Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2015

MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

MATHE1

This question paper consists of 10 pages, including an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.
3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
4. Answers only will not necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. Number the answers correctly according to the numbering system used in this question paper.
8. Write neatly and legibly.
9. An information sheet with formulae is included at the end of the question paper.

QUESTION 1

1.1 Given: $(x+3)(3x-1)=m$

1.1.1 Solve for x if $m=0$. (2)

1.1.2 Solve for x, rounded to two decimal places, if $m=6$. (5)

1.1.3 The turning point of $f(x) = (x + 3)(3x - 1)$ is $(-1\frac{1}{3}; -8\frac{1}{3})$.

(a) How must the graph of f be translated for it to have equal roots? (2)

(b) Hence, write down the values of k for which $f(x) + k = 0$ will have no real solutions. (1)

1.2 Solve for x and y simultaneously in the following equations:

$x-2y=3$
$4x^2-5xy=3-6y$ (6)

1.3 Solve for x if $(3^x-1)(3^x-12)=0$. (4)

1.4 Solve for n if $-n^2+14n+15\geq 0$ (4)

[24]

QUESTION 2

2.1 Given the following arithmetic sequence:

$14\ ;\ 9\ ;\ 4\ ;\ ...$

2.1.1 Determine the value of the 50th term. (3)

2.1.2 Calculate the sum of the first fifty terms. (2)

2.2 The following represents the first three terms of an arithmetic sequence:

$-24\ ;\ p\ ;\ p^2$

Calculate the value(s) of p. (4)

2.3 Consider the geometric series: $3+m+\frac{m^2}{3}+\frac{m^3}{9}+\cdots$

2.3.1 For which value(s) of m will the series converge? (3)

2.3.2 It is given that: $3+m+\frac{m^2}{3}+\frac{m^3}{9}+\cdots=\frac{27}{7}$

Calculate the value of m. (3)

2.4 The sum of the first three terms of a geometric sequence is $31\frac{1}{2}$. The sum of the fourth, fifth and sixth term of the same sequence is $3\frac{15}{16}$. Determine the value of the common ratio (r). (5)

[20]

QUESTION 3

Consider the following number structure:

Row 1	3				
Row 2	6	9			
Row 3	12	15	18		
Row 4	21	24	27	30	
Row 5	33	36	39	42	45

The second term in each row produces the following number pattern:

$9\ ;\ 15\ ;\ 24\ ;\ 36\ ;\ \ldots$

3.1 Determine an expression for the n-th term of the number pattern given above. (4)

3.2 Determine the value of the fifth term in Row 20. (3)

[7]

QUESTION 4

4.1 Patrick opens a savings account on 1 January 2012. He makes an immediate payment of R2 000 into the account and thereafter a monthly payment of R1 200 at the end of each month.
The last payment is made on 31 December 2013. Interest is calculated at 8% per year, compounded monthly.

4.1.1 Calculate the value of Patrick's investment on 31 December 2013. (5)

4.1.2 Patrick decides not to withdraw the money on 31 December 2013. He makes no further payments and the investment earns the same interest rate.

Calculate the value of the investment on 31 May 2014. (3)

4.2 Lilly takes out a loan to the value of R150 000. She repays the loan by means of equal monthly instalments which she makes at the end of each month. The first instalment is made three months after the granting of the loan and the last instalment is eight years after the granting of the loan.
The interest rate is 15% per year, compounded monthly.

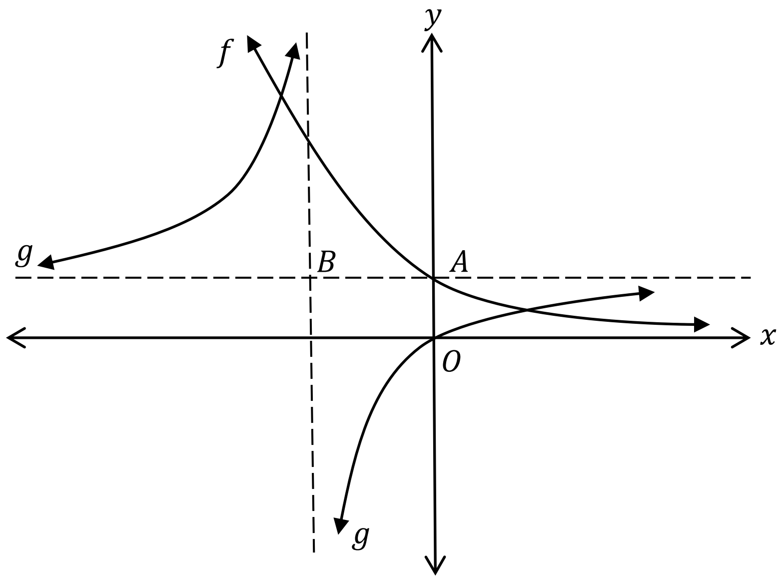
4.2.1 Calculate the value of the equal monthly instalments. (5)

4.2.2 Convert the interest rate to an effective interest rate, rounded to two decimal places. (2)

[15]

QUESTION 5

The sketch shows the graphs of $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \frac{a}{x+p} + q$.
B is the point of intersection of the asymptotes of g. A is the y-intercept of f. The graph of g passes through the origin. AB is parallel to the x-axis.



5.1	Write down the equation of f^{-1} in the form $y = \cdots$.	(2)
5.2	Write down the domain of f^{-1}.	(1)
5.3	Calculate the value(s) of x if $4 \times f(x+1) = \sqrt{2}$.	(3)
5.4	Determine the range of g.	(2)
5.5	If $h(x) = x + 3$ is the equation of one of the axes of symmetry of g, determine the coordinates of B.	(2)
5.6	Hence determine the equation of g.	(4)
5.7	For which value(s) of x is $g'(x) > 0$?	(1)
		[15]

QUESTION 6

Given $f(x) = 2x^2 - 10x - 28$ and $g(x) = mx + k$.

6.1 Write down the y-intercept of f. (1)

6.2 Determine the x-intercepts of f. (3)

6.3 Determine the coordinates of the turning point of f. (2)

6.4 Sketch the graph of f. Clearly show the intercepts with both axes as well as the coordinates of the turning point. (2)

6.5 Determine the coordinates of point P, a point on f, where the gradient of the tangent to f at P is equal to 6. (4)

6.6 Determine the equation of g, the straight line passing through the points $(-2; 0)$ and $(4; -36)$. (3)

6.7 Write down the equation of h in the form $h(x) = a(x + p)^2 + q$ if $h(x) = f(x + 2) - 3$. (3)

[18]

QUESTION 7

7.1 Given: $f(x) =- 5x^2$

Determine $f'(x)$ from first principles. (5)

7.2 Given the following: $y = 8x^3$ and $\sqrt{a} = y^{\frac{2}{3}}$

Determine the following:

7.2.1 $\frac{dy}{dx}$ (1)

7.2.2 $\frac{da}{dy}$ (2)

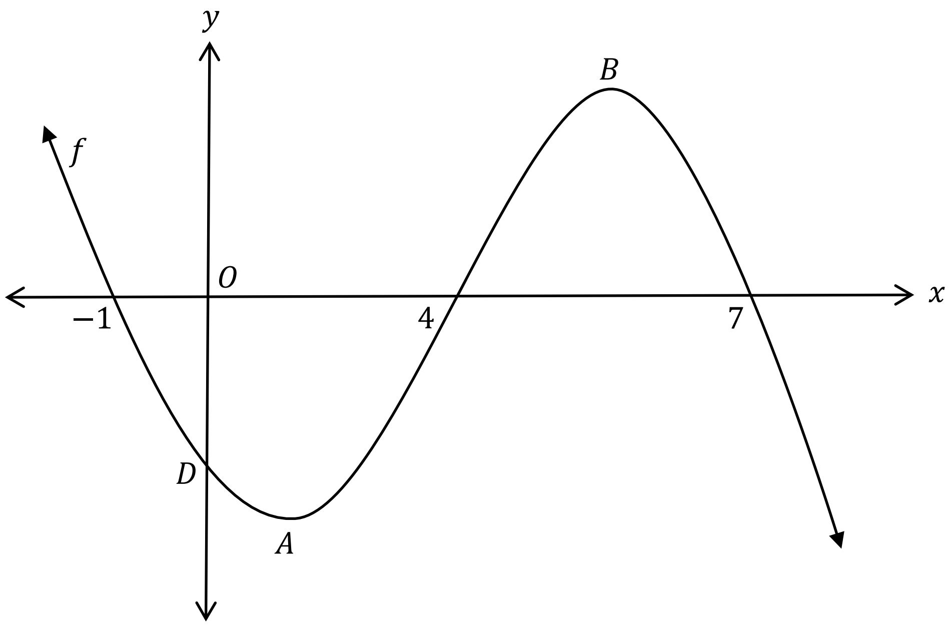
7.2.3 $\frac{da}{dx}$ (3)

7.3 The straight line $g(x) = -8x + 3$ is a tangent to the curve of function f at $x = 5$. Calculate $f(5) - f'(5)$. (3)

[14]

QUESTION 8

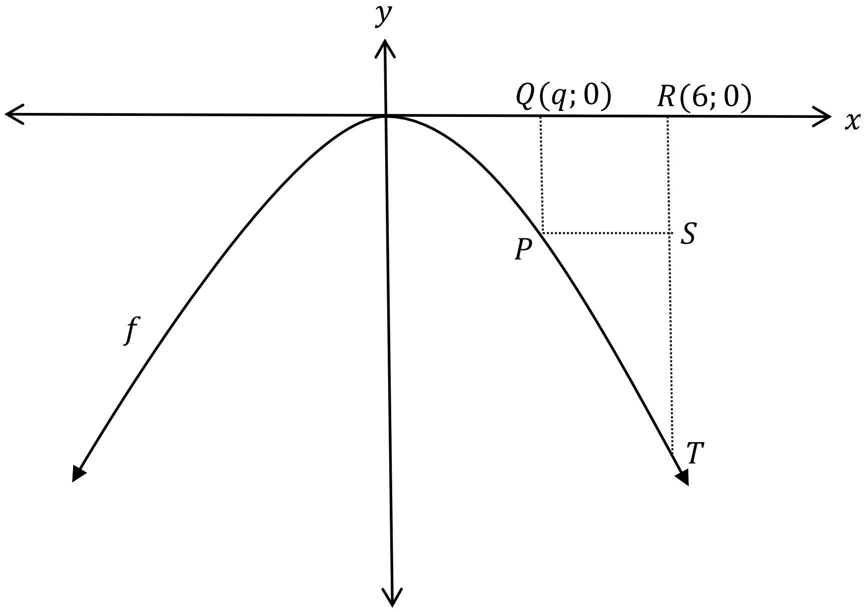
The sketch below shows the graph of $f(x) = -x^3 + 10x^2 - 17x + d$.
The x-intercepts of f are $(-1; 0)$, $(4; 0)$ and $(7; 0)$. A and B are the turning points of f and D is the y-intercept of f. The sketch is not drawn to scale.



8.1	Write down the value of d.	(1)
8.2	Determine the coordinates of A and B.	(5)
8.3	Determine the value of x where the concavity of f changes.	(2)
8.4	Determine the coordinates of the point on f with a maximum gradient.	(2)
8.5	Determine for which value(s) of x is $f(x).f'(x) \geq 0$.	(3)
		[13]

QUESTION 9

The graph below shows the sketch of $f(x) = -2x^2$. R is the point $(6; 0)$ and Q is the point $(q; 0)$. P and T are points on f. RST is parallel to the y-axis and PS is parallel to the x-axis. $PQRS$ is a rectangle.



9.1 Write down the coordinates of P in terms of q. (1)

9.2 Show that the area (A) of rectangle $PQRS$ can be expressed as follows:

$A = 12q^2 - 2q^3$ (2)

9.3 Determine the maximum area (A) of rectangle $PQRS$. (4)

[7]

QUESTION 10

10.1 A and B are two events in a sample space.
$P(not\ A) = 0{,}45$ and $P(B) = 0{,}35$.

10.1.1 Determine $P(A)$. (1)

10.1.2 Determine $P(A\ or\ B)$ if A and B are mutually exclusive events. (2)

10.1.3 Determine $P(A\ and\ B)$ if A and B are independent events. (2)

10.2 A blue (B) and green (G) bucket are filled with balls. The blue bucket contains 5 white (W) and 3 red (R) balls. The green bucket contains 2 white and 7 red balls. A bucket is randomly selected and one ball is thereafter randomly drawn from the bucket.

10.2.1 Draw a tree diagram to represent the above information. Clearly indicate the probability of each branch of the tree. Show all possible outcomes. (4)

10.2.2 Determine the probability that a red ball is drawn. (3)
[12]

QUESTION 11

The Eastern Cape requires new codes for number plates. The new codes consist of four letters followed by four digits, as shown below. All codes end with EC.

BCDF 3856 EC

The vowels (A, E, I, O, U) and Q may not be used and digits 1 to 9 are used. Letters and digits may be repeated.

11.1 Determine how many number plates with different codes can be made. (3)

11.2 Determine the probability that a code that is randomly selected will consist of even digits which are not the same. (2)
[5]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$A=P(1+ni) \qquad A=P(1-ni) \qquad A=P(1-i)^n \qquad A=P(1+i)^n$$

$$T_n=a+(n-1)d \qquad S_n=\frac{n}{2}\left(2a+(n-1)d\right)$$

$$T_n=ar^{n-1} \qquad S_n=\frac{a\left(r^n-1\right)}{r-1}\ ; \quad r\neq 1 \qquad S_\infty=\frac{a}{1-r}\ ;\ -1<r<1$$

$$F=\frac{x\left[(1+i)^n-1\right]}{i} \qquad P=\frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad \mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$$

$$y=mx+c \qquad y-y_1=m(x-x_1) \qquad m=\frac{y_2-y_1}{x_2-x_1} \qquad m=\tan\theta$$

$$(x-a)^2+(y-b)^2=r^2$$

$$In\ \Delta ABC:\quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \qquad a^2=b^2+c^2-2bc.\cos A \qquad area\ \Delta ABC=\frac{1}{2}ab.\sin C$$

$$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta \qquad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta \qquad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases} \qquad \sin 2\alpha=2\sin\alpha.\cos\alpha$$

$$\bar{x}=\frac{\sum fx}{n} \qquad \sigma^2=\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}$$

$$P(A)=\frac{n(A)}{n(S)} \qquad P(A\ or\ B)=P(A)+P(B)-P(A\ and\ B)$$

$$\hat{y}=a+bx \qquad b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$